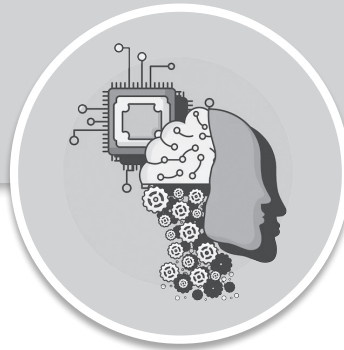


# DATA SCIENCE & ARTIFICIAL INTELLIGENCE

## Calculus and Optimization



Comprehensive Theory  
*with Solved Examples and Practice Questions*





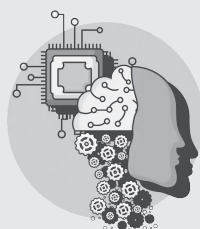
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## Calculus and Optimization

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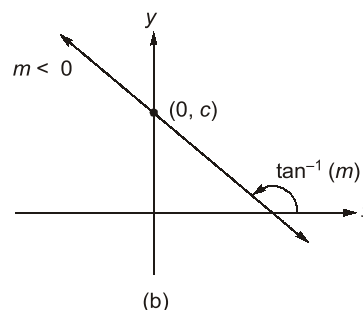
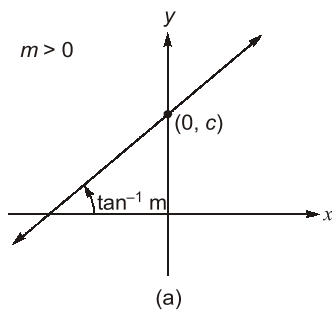
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# Limit, Continuity and Differentiability

## 1.1 GRAPHS OF BASIC FUNCTIONS

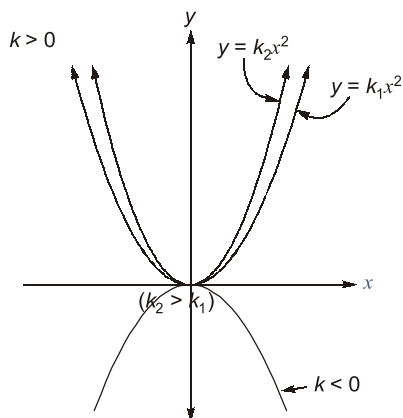
1.



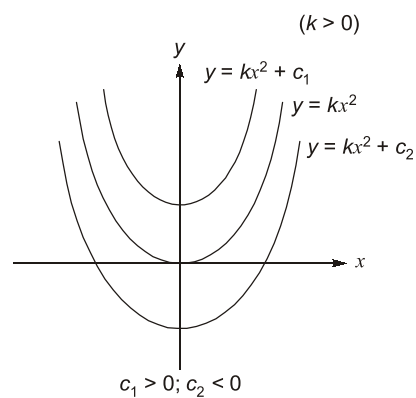
$m \rightarrow$  slope i.e., angle that curve makes with  $x$ -axis in anticlockwise direction.

$c \rightarrow$  intercept i.e., value of  $y$  for  $x = 0$ .

2.  $y = kx^2$   
(similar shape of  $kx^4$ ,  $kx^6$  ....)

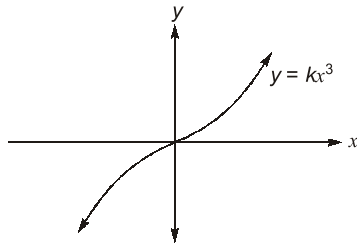


3.  $y = kx^2 + c$



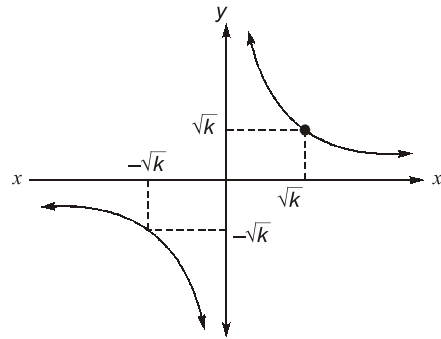
4.  $y = kx^3$

(similar shape for  $kx^5, kx^7 \dots$ )



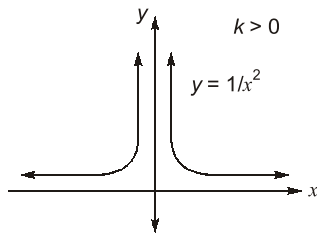
5.  $y = \frac{k}{x}$  or  $xy = k$

(similar shape for  $\frac{k}{x^3}, \frac{k}{x^5} \dots$ )



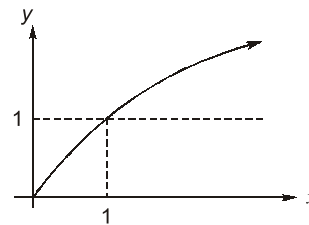
6.  $y = \frac{k}{x^2}$

(similar shape for  $\frac{k}{x^4}, \frac{k}{x^6} \dots$ )

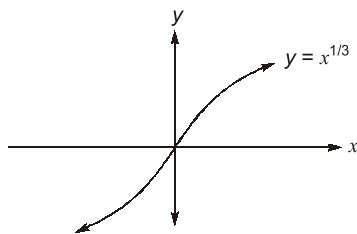


7.  $y = kx^{1/2}$

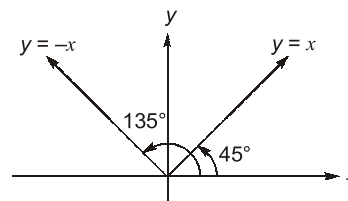
(similar shape for  $kx^{1/4}, kx^{1/6} \dots$ )



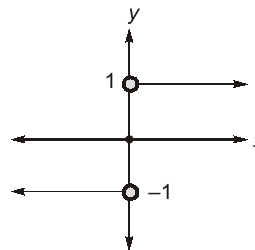
8.  $y = kx^{1/3}$



9.  $y = |x| = \begin{cases} -x & x < 0 \\ 0 & x = 0 \\ x & x > 0 \end{cases}$

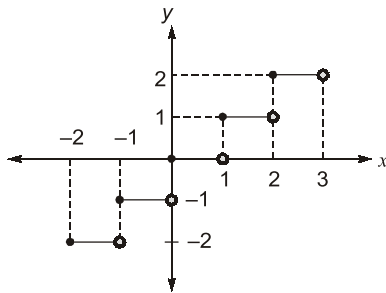


10.  $y = \text{sgn}(x) = \begin{cases} \frac{|x|}{x} & x \neq 0 \\ 0 & \text{else} \end{cases} \rightarrow \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$



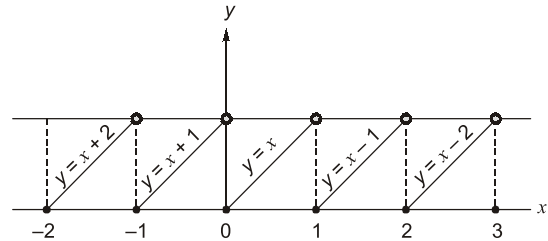
11.  $y = [x]$ , i.e., greatest integer function

$$y = [x] = \begin{cases} \vdots \\ -2 & -2 \leq x < -1 \\ -1 & -1 \leq x < 0 \\ 0 & 0 \leq x < 1 \\ \vdots \end{cases}$$

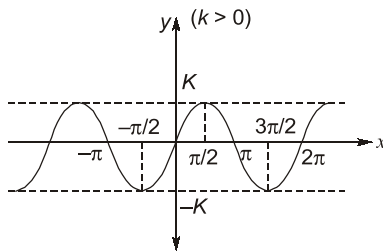


12. Fractional part function

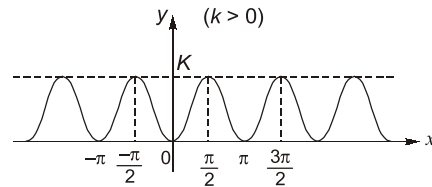
$$y = \{x\} = x - [x] = \begin{cases} \vdots \\ (x+2) & -2 \leq x < -1 \\ x+1 & -1 \leq x < 0 \\ x & 0 \leq x < 1 \\ \vdots \end{cases}$$



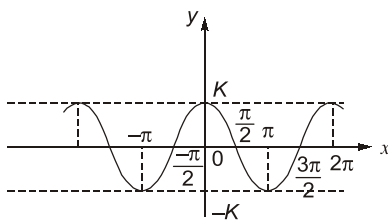
13.  $y = K \sin x$



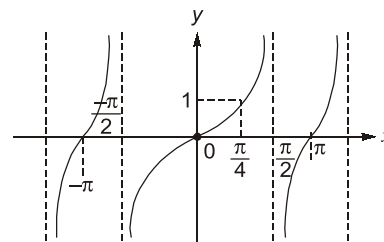
14.  $y = K \sin^2 x$



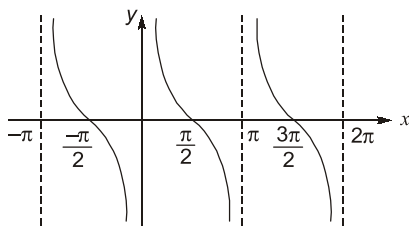
15.  $y = K \cos x$



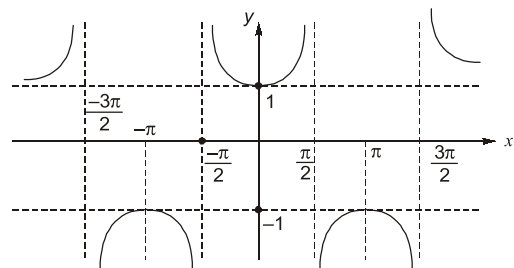
16.  $y = \tan x$



17.  $y = \cot x$



18.  $y = \sec x$




**Student's  
Assignments**

**Q.1** If  $f(x) = \frac{2x^2 - 7x + 3}{5x^2 - 12x - 9}$ , then  $\lim_{x \rightarrow 3} f(x)$  will be

- (a)  $-\frac{1}{3}$                       (b)  $\frac{5}{18}$   
(c) 0                              (d)  $\frac{2}{5}$

**Q.2**  $\lim_{\theta \rightarrow 0} \frac{\sin\left(\frac{\theta}{2}\right)}{\theta}$  is

- (a) 0.5                              (b) 1  
(c) 2                                  (d) Not defined

**Q.3** The Value of  $\lim_{x \rightarrow 8} \frac{x^{1/3} - 2}{(x-8)}$

- (a)  $\frac{1}{16}$                               (b)  $\frac{1}{12}$   
(c)  $\frac{1}{8}$                                   (d)  $\frac{1}{4}$

**Q.4**  $\lim_{x \rightarrow \infty} \frac{x - \sin x}{x + \cos x}$  equals

- (a) 1                                  (b) -1  
(c)  $\infty$                               (d)  $-\infty$

**Q.5** The function  $y = |2 - 3x|$

- (a) is continuous  $\forall x \in R$  and differentiable  $\forall x \in R$   
(b) is continuous  $\forall x \in R$  and differentiable  $\forall x \in R$  except at  $x = \frac{3}{2}$   
(c) is continuous  $\forall x \in R$  and differentiable  $\forall x \in R$  except at  $x = \frac{2}{3}$   
(d) is continuous  $\forall x \in R$  except  $x = 3$  and differentiable  $\forall x \in R$

**Q.6** What should be the value of  $\lambda$  such that the function defined below is continuous at  $x = \frac{\pi}{2}$ ?

$$f(x) = \begin{cases} \frac{\lambda \cos x}{2} & \text{if } x \neq \frac{\pi}{2} \\ \frac{\pi}{2} - x & \\ 1 & \text{if } x = \frac{\pi}{2} \end{cases}$$

- (a) 0                                  (b)  $\frac{2}{\pi}$   
(c) 1                                  (d)  $\frac{\pi}{2}$

**Q.7** The expression  $\lim_{\alpha \rightarrow 0} \frac{x^\alpha - 1}{\alpha}$  is equal to

- (a)  $\log x$                               (b) 0  
(c)  $x \log x$                               (d)  $\infty$

**Q.8**  $\lim_{x \rightarrow 0} \left( \frac{e^{2x} - 1}{\sin(4x)} \right)$  is equal to

- (a) 0                                  (b) 0.5  
(c) 1                                  (d) 2

**Q.9** The value of  $\lim_{x \rightarrow 0} \frac{1 - \cos(x^2)}{2x^4}$  is

- (a) 0                                  (b)  $\frac{1}{2}$   
(c)  $\frac{1}{4}$                                   (d)  $\frac{1}{8}$

**Q.10** The value of  $\lim_{x \rightarrow \infty} (1 + x^2)^{e^{-x}}$  is

- (a) 0                                  (b)  $\frac{1}{4}$   
(c)  $\frac{1}{2}$                                   (d) 1

**Q.11**  $\lim_{x \rightarrow \infty} \sqrt{x^2 + x - 1} - x$  is

- (a) 0                                  (b)  $\infty$   
(c)  $\frac{1}{2}$                                   (d)  $-\infty$

## Explanations

1. (b)

$$\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \left( \frac{2x^2 - 7x + 3}{5x^2 - 12x - 9} \right)$$

Here this is of the form of  $\left(\frac{0}{0}\right)$ .

So, applying L-Hospital's rule

$$\lim_{x \rightarrow 3} \left( \frac{4x-7}{10x-12} \right) = \frac{5}{18}$$

2. (a)

$$\lim_{\theta \rightarrow 0} \frac{\frac{1}{2} \times \sin\left(\frac{\theta}{2}\right)}{\theta \times \frac{1}{2}} = \frac{1}{2} \lim_{\theta \rightarrow 0} \frac{\sin \frac{\theta}{2}}{\frac{\theta}{2}} = \frac{1}{2} = 0.5$$

3. (b)

$$(x-8) = h \text{ (say)}$$

$$\Rightarrow x = 8 + h$$

$$\therefore \lim_{h \rightarrow 0} \frac{(8+h)^{1/3} - 2}{h}$$

Above form in the  $\left(\frac{0}{0}\right)$  by putting the value  $h=0$

Applying L'Hospital rule

$$\lim_{h \rightarrow 0} \frac{\frac{1}{3}(8+h)^{\left(\frac{1}{3}-1\right)}}{1} = \frac{1}{3}(8)^{-\frac{2}{3}} = \frac{1}{12}$$

4. (a)

$$\lim_{x \rightarrow \infty} \frac{x - \sin x}{x + \cos x} = \lim_{x \rightarrow \infty} \frac{1 - \frac{\sin x}{x}}{1 + \frac{\cos x}{x}}$$

$$= \frac{\lim_{x \rightarrow \infty} \left(1 - \frac{\sin x}{x}\right)}{\lim_{x \rightarrow \infty} \left(1 + \frac{\cos x}{x}\right)}$$

$$= \frac{1 - \lim_{x \rightarrow \infty} \frac{\sin x}{x}}{1 + \lim_{x \rightarrow \infty} \frac{\cos x}{x}} = \frac{1-0}{1+0} = 1$$

5. (c)

$$y = |2 - 3x| = 2 - 3x \quad 2 - 3x \geq 0$$

$$= 3x - 2 \quad 2 - 3x < 0$$

$$\text{Therefore, } y = 2 - 3x \quad x \leq \frac{2}{3}$$

$$= 3x - 2 \quad x > \frac{2}{3}$$

Since  $2 - 3x$  and  $3x - 2$  are polynomials, these are continuous at all points. The only concern is

$$\text{at } x = \frac{2}{3}$$

$$\text{Left limit at } x = \frac{2}{3} \text{ is } 2 - 3 \times \frac{2}{3} = 0.$$

$$\text{Right limit at } x = \frac{2}{3} \text{ is } 3 \times \frac{2}{3} - 2 = 0.$$

$$f\left(\frac{2}{3}\right) = 2 - 3 \times \frac{2}{3} = 0$$

$$\text{Since, Left limit} = \text{Right limit} = f\left(\frac{2}{3}\right),$$

Function is continuous at  $\frac{2}{3}$ .

$y$  is therefore continuous  $\forall x \in R$

Now since  $2 - 3x$  and  $3x - 2$  are polynomials, they are differentiable.

$$\text{Only concern is at } x = \frac{2}{3}.$$

$$\text{Now, at } x = \frac{2}{3}, \text{ LD} = \text{Left derivative} = -3$$

$$\text{RD} = \text{Right derivative} = +3$$

$$\text{LD} \neq \text{RD}$$

$$\therefore \text{The function } y \text{ is not differentiable at } x = \frac{2}{3}$$

So, we can say that  $y$  is differentiable  $\forall x \in R$ ,

$$\text{except at } x = \frac{2}{3}.$$

6. (c)

If  $f(x)$  is continuous at  $x = \frac{\pi}{2}$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\lambda \cos x}{\frac{\pi}{2} - x} = f\left(\frac{\pi}{2}\right) = 1 \quad \dots(i)$$